Intro to Standard Model

Here is a bunch of data:

Br

$D \to K \mu \nu = 3.33\%$

$D \to \pi \mu \nu = 2.38 \cdot 10^{-3}\%$

$B \to X \mu \nu = 10.86\%$

$B \to X e \nu = 10.86\%$

$B \to D l \nu = 2.2 \cdot 10^{-2}$

$B \to \pi l \nu = 1.5 \cdot 10^{-4}$

$a_{cp}(K_L \to \pi l \nu) = 3.32 \cdot 10^{-3}$

$a_{cp}(B \to K^+ \pi^-) = -0.082$

$B_s \to \mu^+ \mu^- = 3 \cdot 10^{-9}$

$K_L \to \mu^+ \mu^- = 7 \cdot 10^{-9}$

$D \to \mu^+ \mu^- < 6 \cdot 10^{-9}$

$J/\psi \to \mu^+ \mu^- = 6\%$

$K^+ \to \mu^+ \nu = 0.64$

$\mu \to e \gamma < 4.2 \cdot 10^{-13}$

$d_e < 10^{-29}$ e.cm

p decay $> 10^{34}$ y

n decay $= 880$ sec

$Z \to ee, \mu\mu, \tau\tau = 3.3\%$

$Z \to$ hadrons $= 70\%$

$Z \to \gamma\gamma < 1.2 \cdot 10^{-5}$

$W \to e\nu, \mu\nu, \tau\nu = 10.8\%$

$W \to$ hadrons $= 67\%$

This is Nature, together with the 800+ pages of the PDG, full of particles with their measured decay rates, etc.

Alredy just staring at the data we can realize a few facts:

- Neutron decays, proton doesn't.
- $\mu \to e\gamma$ unseen
- e^- EDM unseen
- lepton universality
- $2\to1 > 3\to2 > 3\to1$ transitions
- FCNC small

So we immediately see a few qualitative features. It might seem hard to find a framework that allows to explain, qualitatively, all 800+ pages at the same time.

One might even think that having a quantitative understanding of all 800+ pages is beyond scope, even more crazy would be to expect that a simple structure behind it.

Well, here it is:

$$\mathcal{L} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^{a}_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}G^{A}_{\mu\nu}G^{A\mu\nu} + i\bar{\psi}\not{D}\psi \quad \Big] \text{ gauge}$$

$$+ y_{ij} H \bar{\psi}_{Li}\psi_{Rj} - \frac{1}{2}\mu^2 H^2 - \lambda H^4 \quad \Big] \text{ Higgs}$$

It is trully remarkable that these two lines can capture, accurately, the physics of all phenomena observed at colliders, and at nuclear & atomic exp.

For instance, quite famously, the electron g-2 is

$$\text{th}: \quad a_e = 0.00115965218164\,3(74)$$

$$\text{exp}: \quad a_e = 0.00115965218059(13)$$

$\updownarrow$ α_{em} from Cs/Rb generates discrepancy.

This success comes from the 'gauge' part of the Lagrangian above.

Part of this course is about gauge theories, what they are, how to construct them. You will see that they are very constrained.

The Higgs part of the Lagrangian is less constrained, it leads to all the 'messiness' of Nature, and every single piece that composes it is shrd with mistery.

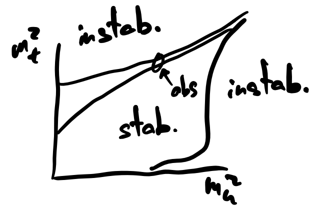
- y_{ij} could've been any matrix, but experimentally is $\sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$ with $\lambda \sim 0.2$.

<u>why?</u> No one knows

- There is no symm protecting μ^2, so it must be $\propto \frac{y^2}{16\pi^2}\Lambda^2$, with Λ being the next scale of the theory. If $\Lambda = M_{PL}$, this is completely wrong.

  why? No one knows

- λ is such that universe is in a meta-stable state



  why? No one knows.

To these questions, one should add the total failure to explain other facts of nature:

- Dark Matter, dark energy, matter antim. asymm, etc.

So the SM is our explanation of Nature, it works remarkably well in describing collider data, some cosmological data (e.g. abundances of elements), but it is not <u>the</u> theory of the world.

GAUGE THEORIES

In order to describe QFT's with massless vectors, we use a gauge theory.
A gauge theory is a QFT with a <u>gauge-invariant</u> Lagrangian. The simplest example is QED.

We describe the photon by a vector field A_μ, which has associated the gauge transformation

$$A_\mu(x) \longrightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu \alpha(x) \quad , \quad \alpha \in \mathbb{R}$$

Why this transformation should be obeyed by the vector field will be understood after Riccardo's lectures. For now, we can take this as the defining property of (abelian) gauge theories.

The invariant object under gauge transformations is the field strength,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

With this, the unique choice for a gauge invariant kinetic term is

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} .$$

You can check that $F_{\mu\nu} F_{\nu\rho} F_{\rho\mu}$ vanishes.

The most relevant operator (in the sense of the one with smallest scaling dimension) we can write is

$$\mathcal{L} \supset \frac{1}{\Lambda^4} (F_{\mu\nu} F_{\mu\nu})^2$$

We will discuss later operators with dim > 4. We refer to them as 'irrelevant' in the sense that their effect decreases at low energies.

So we want to define QED as the Lagrangian made out of the "renormalizable" terms. The name is terrible, since "non-renormalizable" theories are renormalizable.

Therefore, we conclude that the photon does not selfinteract.

Now we add matter, i.e. electrons. These are made out of spin-1/2 Dirac fermions.

The operator with the lowest dimension we can write is

$$\mathcal{L} \supset \frac{1}{\Lambda} \bar{\psi} \sigma_{\mu\nu} \psi F^{\mu\nu}$$

but it is dim 5. So, irrelevant, and for the moment we throw it away. What do we do then?

We postulate that the matter fields are "charged" under gauge transformations,

$$\psi(x) \longrightarrow \psi'(x) = e^{ie\alpha(x)} \psi(x)$$

Here e is just a free parameter.

Given that this is a local transformation, the term $\partial_\mu \psi$ does not transform properly, but the combination

$$(\partial_\mu - ieA_\mu)\psi \equiv D_\mu \psi$$

does,

$$\rightarrow (\partial_\mu - ie(A_\mu + \partial_\mu \alpha))\, e^{ie\alpha}\psi = e^{ie\alpha(x)}(\partial_\mu + ie\partial_\mu\alpha - ieA_\mu - ie\partial_\mu\alpha)\psi$$

$$= e^{ie\alpha(x)} D_\mu \psi .$$

This leads to the QED Lagrangian

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \not{D} \psi - m \bar{\psi}\psi$$

- This is called abelian theory since matter transforms as a $U(1)$ rep. So gauge group is $U(1)$.

- The photon is massless by construction, imposing $A_\mu \to A_\mu + \partial_\mu \alpha$. In some textbooks the logic is reversed: One imposes a gauge transf on matter, then introduces a photon and associates a transformation for consistency. Then $m_\gamma^2 A_\mu A^\mu$ is forbidden by gauge invariance.

- Minimal coupling of photon to electron.
- No photon self-interaction.
- The equations of motion for A_μ,

$$\partial_\nu F^{\mu\nu} = J^\nu \qquad , \quad J^\nu = \bar{\psi}\gamma^\nu\psi$$

are the first two Maxwell equations. Note that $\partial_\mu\partial_\nu F^{\mu\nu} = 0$, which implies $\partial_\mu J^\mu = 0$ is conserved.

- The two other Maxwell eq can be written as

$$\epsilon^{\mu\nu\rho\sigma}\partial_\nu F_{\rho\sigma} = 0,$$

which is automatically satisfied.

- Feynman rule  $= -ie\gamma^\mu$

Ward identity.

The Ward identity guarantees that the unphysical polarizations of the photon are not produced in a scattering process.

For the case of a massive vector boson, little group rotations mix polarizations with themselves.

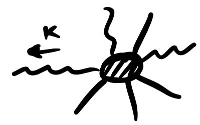
$$\varepsilon^\mu_\lambda \longrightarrow \varepsilon^{\mu\,\prime}_\lambda = R^\mu{}_\nu\, \varepsilon^\nu_\lambda = \sum_{\lambda'} c_{\lambda\lambda'}\, \varepsilon^\mu_{\lambda'}$$

This is not true for the massless little group, since ISO(2) translations shift the polarization vectors by the photon's momenta,

$$\varepsilon^\mu_\lambda \longrightarrow \varepsilon^{\mu\,\prime}_\lambda = \varepsilon^\mu_\lambda + \alpha\, p^\mu$$

This has the same origin as the $A_\mu + \partial_\mu \alpha$ redundancy. Therefore, this shift must be annihilated in any physical observable.

In particular, take an amplitude with an ext. photon,

$$T = \varepsilon^\mu_\lambda(k)\, \mathcal{M}_\mu = \varepsilon^\mu$$ 

Then, it must be that

$$p^\mu \mathcal{M}_\mu = 0 .$$

This is the Ward identity.

- We didn't show it is true, just that it must hold in any reasonable theory.

For instance, since the gauge coupling comes from the kinetic term, it is diagonal, does not mix

different fermions.
In the exercise you will encounter an 'unreasonable' theory.

- **Anomalous magnetic moment**

The Dirac equation becomes

$$(i\partial\!\!\!/ + eA\!\!\!/ - m)\psi = (iD\!\!\!\!/ - m)\psi = 0$$

we can compare it with the one of a scalar field,

$$(D_\mu^2 + m^2)\phi = 0$$

To do so, multiply the above by $i\partial\!\!\!/ + eA\!\!\!/ + m$, giving

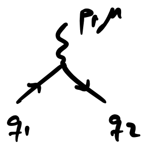
$$(D_\mu^2 + m^2 + \frac{e}{2}F^{\mu\nu}\sigma_{\mu\nu})\psi = 0.$$

$F_{\mu\nu}$ shows up from $D\!\!\!\!/^2$, and using $[D_\mu, D_\nu] = ieF_{\mu\nu}$.

In the presence of a magnetic field, this leads to a $ge\vec{B}\cdot\vec{S}$ coupling with the electron's spin.
For spin 1/2, then $g=2$.
This was a huge success at the time, but by late 1940's experiments were suggesting $g-2 \neq 0$, so a small anomalous magnetic moment.
The quantity receives quantum corrections:

The off-shell matrix element

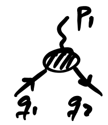
$i\mathcal{M}_0^\mu =$



$$= -ie\,\bar{u}(q_2)\gamma^\mu u(q_1) \quad \text{w/ } p^\mu = q_1^\mu - q_2^\mu$$

This form is "a tree level" accident and it is modified by loops. The general form

$i\mathcal{M}^\mu =$



$$= \bar{u}(q_2)\left(f_1\gamma^\mu + f_2 p^\mu + f_3 q_1^\mu + f_4 q_2^\mu\right)u(q_1)$$

Using momentum conservation we can chose to set $f_2 = 0$. The ward identity $p^\mu \mathcal{M}_\mu$ further removes another parameter. Only two remain, which can be written as

$$i\mathcal{M}^\mu = (-ie)\,\bar{u}(q_2)\left[F_1\left(\frac{q^2}{m^2}\right)\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m}p_\nu F_2\left(\frac{q^2}{m^2}\right)\right]u(q_1)$$

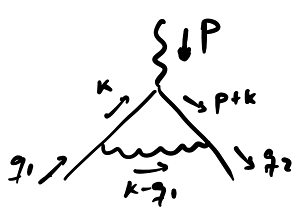
the F's are form factors, and at leading order

$$F_1 = 1\,, \qquad F_2 = 0\,.$$

F_1 modifies the coupling e, so any contribution to the g-2 comes from F_2,

$$\frac{g-2}{2} = F_2(0)$$

We want to evaluate the $p_\nu\sigma^{\mu\nu}$ contribution from



$$i\mathcal{M}^\mu = (-ie)^3 \int \frac{d^4k}{(2\pi)^4} \frac{-i\eta^{\nu\alpha}}{(k-q_1)^2+i\varepsilon} \bar{u}(q_2)\gamma^\nu \cdot \frac{i(\not{p}+\not{k}+m)}{(p+k)^2-m^2+i\varepsilon}\gamma^\mu$$

$$\cdot \frac{i(\not{k}+m)}{k^2-m^2+i\varepsilon}\gamma^\alpha u(q_1)$$

$$= -e^3\,\bar{u}(q_2)\int\frac{d^4k}{(2\pi)^4}\frac{\gamma^\nu(\not{p}+\not{k}+m)\gamma^\mu(\not{k}+m)\gamma_\nu}{[(k-q_1)^2+i\varepsilon][(p+k)^2-m^2+i\varepsilon][k^2-m^2+i\varepsilon]}u(q_1)$$

You will learn how to deal with loop integrals in Advanced QFT.

1) Feynman parameters

$$\frac{1}{ABC} = 2\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\frac{1}{(xA+yB+zC)^3}$$

2) Wick rotate

$$k^0 \to ik^0 \Rightarrow k^2 = -k_E^2$$

3) Most importantly, what the infinities of the above integral mean & what to do with them.

It turns out all divergent part of the integral is related with F_1, and has to do with the running of the electromagnetic coupling.

F_2 contribution is finite (check P&S or Schwartz 17.2)

$$F_2(p^2) = \frac{\alpha}{\pi} m^2 \int_0^1 dx\,dy\,dz\, \delta(x+y+z-1) \frac{z(1-z)}{(1-z)^2 m^2 - xy\, p^2}$$

$$\Rightarrow \quad F_2(0) = \frac{\alpha}{\pi} \int_0^1 dz \int_0^{1-z} dy\, \frac{z}{1-z} = \frac{\alpha}{2\pi}$$

So $\quad g-2 = \frac{\alpha}{\pi} = 0.00232\ldots$